INVESTIGATION OF TWO 2D PROPELLER CALCULATION METHODS AND THE DEPENDENCY OF THE SOLUTION IN RELATION TO THE NUMBER OF CALCULATION POINTS

Jakub HNIDKA, Bang Van BANG, Dalibor ROZEHNAL

Faculty of Military Technology, University of Defence, Brno, Czech Republic
(jakub.hnidka@unob.cz, nguyen.vanbang@unob.cz, dalibor.rozehnal@unob.cz)

DOI: 10.19062/2247-3173.2018.20.23

Abstract: *This paper presents two simple implementations of blade element method used for propeller performance calculation – Blade Element Momentum Theory and Joukowsky method based on a simple vortex model. In principle, both methods require division of the blade of the propeller into finite number of sections (i.e. calculation points), over which a 2D flow is assumed. The focus of this paper is to investigate the dependency of the calculated solution on the number of sections and comparison of both methods. For this purpose, three propeller geometries were chosen and a simple method used for comparison was developed. The calculation was performed in LabVIEW interface implementing MATLAB code.*

Keywords: *Blade Element Momentum Theory, Joukowsky method, propeller performance, LabVIEW*

1. NOMENCLATURE

B [1]	Number of Blades	
c [m]	Local chord length	
$\bar{c}$ [1]	Relative chord length	$\bar{c}=cD$
$\bar{r}$ [1]	Dimensionless radius	$\bar{r}=rR$
t [m]	Local thickness	
$\bar{t}$ [1]	Dimensionless thickness	$\bar{t}=tc$
U1 [ms-1]	Angular flow velocity vector	
V1 [ms-1]	Axial flow velocity vector	

2. INTRODUCTION

With advances of the unmanned aerial vehicles (UAVs) and new technologies to manufacture propellers, a need arises to quickly and efficiently calculate the propeller characteristics. While it is possible e.g. to implement a CFD calculation to obtain precise results, the tuning of the calculation is challenging and it does take considerable time [1].

Blade element methods (BEM), while conceptually simple, provide sufficiently precise results and due to their relative simplicity, they are widely used not only to perform the initial aerodynamic analysis, but also to optimize the design and calculate the aerodynamic loads acting on the propeller. These methods are also suitable to calculate the performance of the wind turbines. [2]

This paper presents the more traditional Blade Element Momentum Theory (BEMT), which couples the classical BEM with the momentum theory and Blade Element Joukowsky Method, which implements circulation distribution calculation along the rotor blade based on the Joukowsky theorem. Both methods have fundamentally the same assumptions [3]:

- The rotor can be represented by a finite amount of sections and the flow around each section can be approximated by a 2D flow around the airfoil
- The rotor is lightly-loaded
- The wake behind the rotor is cylinder-like
- The inflow is axisymmetric to the axis of rotation (zero yaw angle)
- The rotor geometry is known
- The airfoil polar in each calculated section is known
- The blades of the propeller are infinitely stiff

This paper closely inspects the influence of the number of calculation points given by the rotor division to the solution. For this purpose, three different rotor geometries were assumed and the number of sections (or calculation points) varies from 10 to 60. The overall propeller characteristics are observed and compared.

3. INPUT DATA

3.1 Rotor Geometries. Three rotor geometries were assumed. The first rotor is a helicopter rotor with a simple flat plate geometry. The second one is a modeler rotor from a propeller model aircraft. The third one is from a small propeller aircraft. The rotor geometries are shown in the Fig. 1.

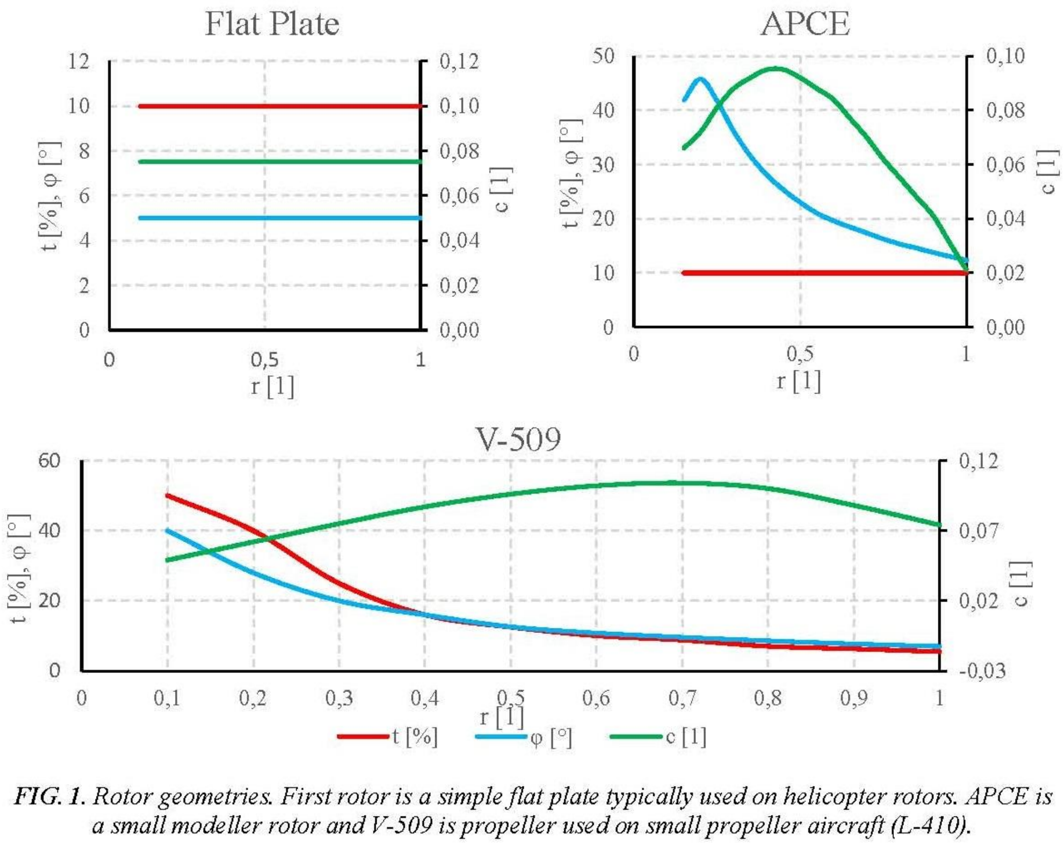

FIG. 1. Rotor geometries. First rotor is a simple flat plate typically used on helicopter rotors. APCE is a small modeller rotor and V-509 is propeller used on small propeller aircraft (L-410).

3.2 Airfoil Data. The rotor is then divided into finite number of sections, in which 2D flow over an airfoil is assumed. In order to perform the calculation via blade element method, the airfoil polar has to be known. Typically, the airfoil data can be measured or calculated. The measured data can be obtain e.g. from NTRS servers [4]. However, it is very difficult to obtain the measured polar for all Reynolds numbers and all airfoil thicknesses achieved along the rotor radius.

It is possible to calculate the polar e.g. in CFD or use other analytical methods. For purposes of this article, a paneling method developed by Drela and implemented in XFOIL was used [5].

XFOIL can be used to obtain precise data in pre-stall regimes. The iterative solution of BEMT or Joukowsky method can lead to values of angle of attack (AoA) greater than a stall angle. It is necessary to extrapolate the calculated polar to full ±180°. The extrapolation was performed by Viterna-Corrigan method further described in [6]. Comparison of extrapolated data calculated in XFOIL, CFD and measured data is described in [7] and an example of the airfoil polar is shown in Fig. 2.

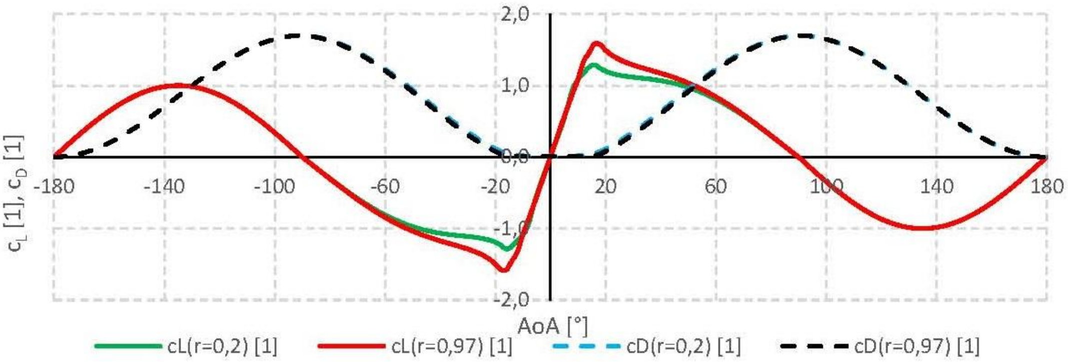

FIG. 2. The lift and drag polar for NACA 16 for first and last element calculated in XFOIL and extrapolated by Viterna-Corrigan

Snel [8] and subsequently Selig [9] showed that the assumption of 2D polar leads to underprediction of propeller performance due to the rotational effects influencing the flow over the airfoil. The rotational effects cause a shift in the stall angle into higher values of local AoA. However, for purposes of this paper these corrections were not implemented.

4. BLADE ELEMENT METHODS

Both Blade Element Momentum Theory and modified Joukowsky theorem implement Blade Element Method, while combining it with Momentum theory, or Joukowsky theorem, respectively.

4.1 Blade Element Method. The first step of the BEM theory is the subdivision of the blade into the finite amount of blade elements, as shown in the Fig. 3.

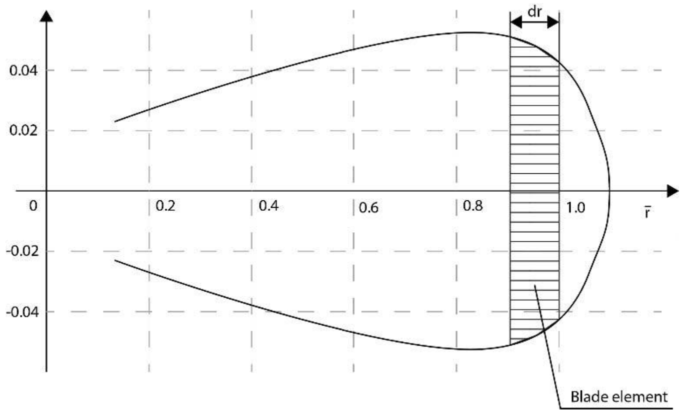

FIG. 3. The subdivision of rotor into finite amount of blade elements

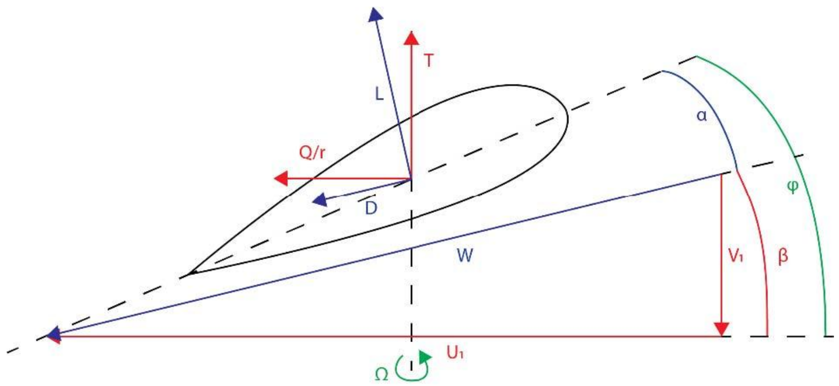

FIG. 4. The local element velocities and angles

In order to quantify the element aerodynamic properties, it is necessary to know the local angle of attack (AoA) α. Based on the Fig. 4 the AoA is equal to the difference of the geometric pitch angle φ and a local inflow angle β:

$$\alpha=\varphi-\beta$$

Assuming the blade motion is sufficiently small, the inflow angle β is then equal to:

$$\beta = \tan^{-1}\frac{V_1}{U_1}$$

The local lift L and drag D of the element can be calculated as:

$$\mathrm{d}L = c_L \frac{1}{2}\rho W^2 c\mathrm{d}r$$

$$\mathrm{d}D = c_D \frac{1}{2}\rho W^2 c\mathrm{d}r$$

The thrust dT and the torque moment dQ produced by B blades can be obtained by a simple trigonometry based on the Fig. 2:

$$\mathrm{d}T = \frac{1}{2}\rho W^2 c(c_L \cos\beta - c_D \sin\beta)B$$

$$\mathrm{d}Q = \frac{1}{2}\rho W^2 cr(c_L \cos\beta - c_D \sin\beta)B$$

4.2 Momentum Theory. According to the momentum theory, the thrust produced by the propeller is equal to change in the momentum of the flow passing through the propeller disk and the torque is equal to change in the angular momentum of the flow and radius:

$$\mathrm{d}T = 2\pi r\rho W(V_w - V)\mathrm{d}r$$

$$\mathrm{d}Q = 2\pi r\rho W(2a'\Omega r - 0)\mathrm{d}r$$

where V_w is the velocity in the far wake, r is local radius and is the rotational augmentation factor. It can be shown that:

$$V_w = V(1 + 2a)$$

$$W = \frac{V + V_w}{2} = V(1 + a)$$

where is the tangential augmentation factor. After few simple rearrangements, the thrust and torque can be expressed as:

$$\mathrm{d}T = 4\pi r \rho V^2 (1 + a)\mathrm{d}r$$

$$\mathrm{d}Q = 4\pi r^3 \rho V(1 + a)a'\Omega \mathrm{d}r$$

By rearrangement of the aforementioned equations, we can come to an iterative process used to calculate the rotational and tangential augmentation factors a' and a.

4.3 Joukowsky Theorem. In order to correctly calculate the values of velocities and , Joukowsky theorem can be used. The flow over the blade section is approximated by the dimensionless circulation , which can be expressed as:

$$\bar{\Gamma} = \frac{B}{4\pi} c_L \bar{c} \frac{U_1}{R\Omega \cos\beta}$$

The velocities and can be expressed as the function of the dimensionless circulation :

$$U_1 = \left(\bar{r} - \frac{\bar{\Gamma}}{\bar{r}}\right) R\Omega$$

$$V_1 = \left(\frac{\lambda}{2\pi} + \sqrt{\left(\frac{\lambda}{2\pi}\right)^2 + \bar{\Gamma}(\bar{\Gamma} - 1)}\right) R\Omega$$

By combining the aforementioned equation with Blade Element Theory, it is possible to calculate the dimensionless circulation by an iterative process. The thrust and torque of the blade section are then equal to:

$$dT = \frac{1}{4} B\bar{c}\rho D^2 \Omega \frac{U_1}{\cos\beta}(c_L U_1 - c_D V_1)$$

$$dQ = D^2 \pi^2 r B\bar{c} \frac{U_1}{\cos\beta}(c_L V_1 + c_D U_1)$$

4.4 Tip loss problem. Figure 5 shows the distribution of the circulation along the dimensionless radius as calculated by the Jouowsky theorem. Similar distribution can be obtained by BEMT. The circulation was not calculated for , because the propeller in this region does not produce any thrust. However, because both theories are derived for azimuthally independent stream tubes, they are only valid for infinitely many blades [10]. Prandtl showed that for a finite blade, due to the pressure equalization between upper and lower parts of the blade at its tips, the produced lift (and subsequently circulation) is equal to zero [11]. Glauert [12] derived Prandlt's tip loss factor for BEMT. While typically a Prandtl's tip loss model is implemented for wind turbine calculations, e.g. in [13] it is used for propeller calculation. A similar tip loss factor was derived for Joukowsky theorem.

For purposes of this paper, a simplest tip-loss factor was assumed. The thrust of the propeller is proportional to the integral of the circulation.

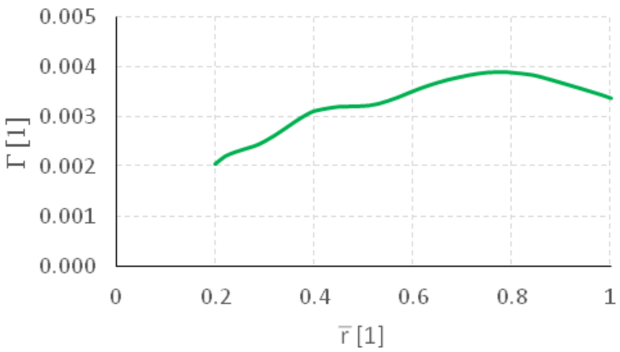

FIG. 1. The distribution of the dimensionless circulation along the without the tip loss model assumed

4.5 Performance coefficients. The propeller performance is typically expressed as a function of thrus $\bar{r} < 0.97$ower coefficie and efficiency. The coefficients can be calculated as:

$$c_T = \frac{T}{\rho n^2 D^4}$$

$$c_N = \frac{N}{\rho n^3 D^5} = 2\pi c_Q$$

$$\eta = \frac{c_T}{c_N}\lambda = \frac{c_T}{2\pi c_Q}\lambda$$

Typical propeller performance characteristics are shown in the Fig. 6.

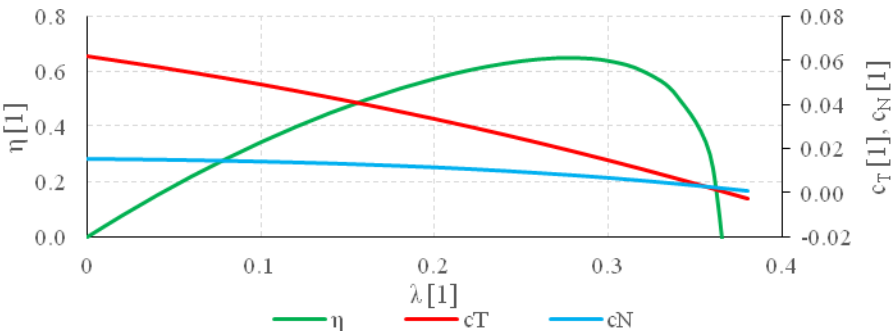

FIG. 2. Calculated propeller performance characteristics of a third propeller

5. CONVERGENCE INVESTIGATION

The propeller performance characteristics, as shown in the Fig. 6, were calculated for diffrent number of calculation points varying from 10 to 60. The upper limit was chosen due to the time load of the calculation itself – with 60 calculation points, it is necessary to actually calculate 60 diffrent airfoil polars. The comparison was performed followingly:

$$\eta_{\%} = \left|\frac{\eta_n - \eta_{n+10}}{\eta_{n+10}}\right| 100\ [\%]$$

where n is the number of calculation points. Identical calculation was performed for c_T and c_N. For a convergent process, following should be applicable:

$$\lim_{n\to\infty} \eta_{\%} = 0$$

However, if the limit starts converging only for large values of *n* (*n*>100), then the blade element method is time consuming and on par with more sophisticated analytical methods (3D paneling methods).

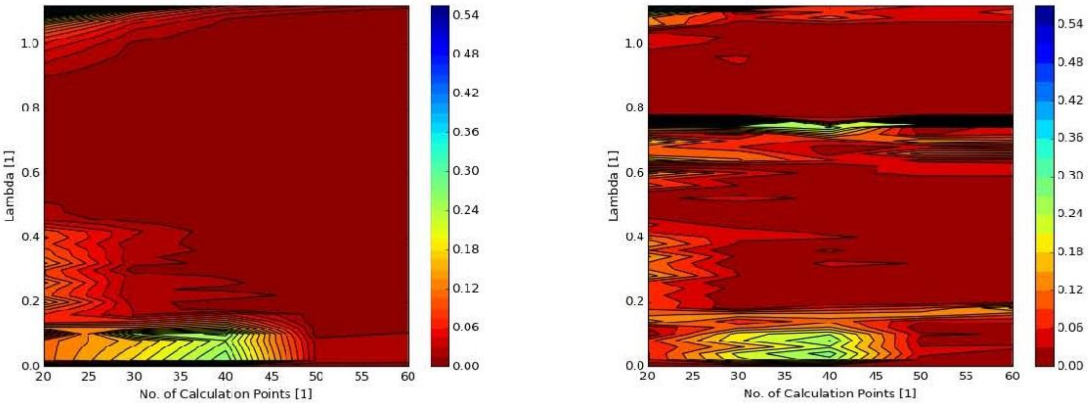

FIG. 3. Comparison of $\eta_{\%}$ calculated for second propeller. Joukowsky (left) and BEMT (right) (APCE)

Fig.7. displays values of $\eta_{\%}$ calculated for second geometry assumed. Both BEMT and Joukowsky method show considerable deviations for low values of , i.e. in static regime.

The apparent disruption of the BEMT result for between 0.6 and 0.8 was observed only for this particular geometry.

Further disruptions are observed at large values of for BEMT. This regime corresponds to the regime of zero thrust. Typically, the propeller aircraft can enter this zone only under very specific conditions (e.g. nose dive) and it is not important for calculation of a propeller performance. Figure 7. shows the worst results achieved – for other two geometries performed both methods more similar.

In order to determine the optimum amount of calculation points along the radius, the change of the calculated values compared to final (*n* = 60) was investigated as well:

$$\eta^*_{\%} = \eta_n - \eta_{60}[1]$$

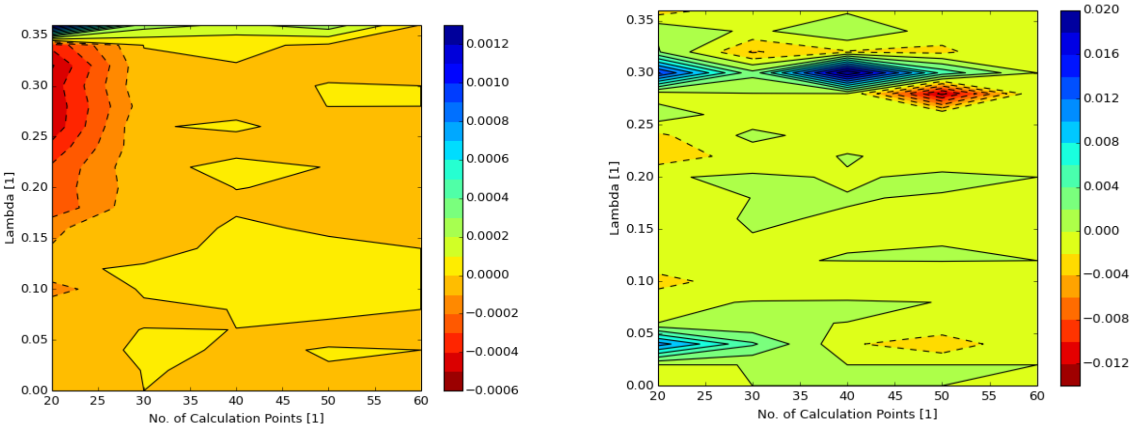

FIG. 4. Comparison of $\eta_{\%}$ calculated both for Joukowsky (left) and BEMT (right) (V509)

Figure 8. pictures the dependency of $\eta_{\%}$ on number of calculation points. The Joukowsky method performed considerably better even for as low as 20 calculation points. However, BEMT requires more calculation points to better approximate the results close to the maximum propeller efficiency.

6. Conclusion

The paper presents two blade element methods widely used to calculate the propeller performance. It explains in detail the differences in both implementations and then investigates the dependency of the solution obtained on the number of calculation points.

Considering the application of blade element methods in the initial phases of design of a new propeller, both versions perform exceptionally well. Nonetheless, for all examples tested, the Joukowsky method performs better and produces more stable solutions. Authors do recommend to check the solution by simply repeating the calculation again with larger amount of calculation points.

Both methods produce quick and precise results – however, a lot of calculation time is spent in the pre-processing phase, where airfoil polar for each blade section has to be calculated. For purposes of this paper, XFOIL was implemented into the program written marginally in LabVIEW and partially in MATLAB.

For this comparison, only axial flow was considered. However, both methods can be adjusted to implement also the yaw angle. In future, both the modification and the dependency of the solution on the azimuthal division will be presented.

7. Acknowledgements

This research was conducted in cooperation with company ENERGOKLASTR.

References

[1] A. Kusyumov, S. Kusyumov, S. Mikhailov, E. Romanova, K. Phayzullin and E. a. B. G. Lopatin, "Main rotor-body action for virtual blades model," in *Experimental Fluid Mechanics*, Liberec, 2017;

[2] S. Ning, G. Hayman, R. Damiani and J. Jonkman, "Development and Validation of a New Blade Element Momentum Skewed-Wake Model within AeroDyn," in *AIAA Science and Technology Forum and Exposition*, Kissimmee, 2015;

[3] V. L. Alexandrov, Letecké vrtule, Praha: Státní Nakladatelství Techincké Literatury, 1954;

[4] NASA, "NASA Technical Report Server," NASA, [Online]. Available: https://ntrs.nasa.gov/.

[5] M. Drela, "XFOIL: An Analysis and Design System for Low Reynolds Number Airfoils," in *Low Reynolds Number Aerodynamics*, Springer-Verlag, 1989;

[6] L. Viterna and R. Corrigan, "Fixed Pitch Rotor Performance of Large Horizontal Wind Turbines," NASA, Cleveland, 1995;

[7] D. Rozehnal, V. B. Nguyen and J. Hnidka, "Hysteresis Assumptions for Propeller in Yaw in Close Stall Conditions," *Engineering Mechanics,* 2018;

[8] H. Snel, R. Houwink, G. van Bussel and A. Bruining, "Sectional Prediction of 3D Effects for Stalled Flow," *European Community Wind Energy,* pp. 395-399, 8-12 March 1993;

[9] D. V. Uhlig and M. Selig, "Post Stall Propeller Behavior at Low Reynolds Numbers," *46th AIAA Aerospace Sciences Meeting and Exhibit,* 7-10 January 2008;

[10] V. L. Okulov, J. N. Sørensen and D. H. Wood, "The rotor theories by Professor Joukowsky: Vortex theories," *Progress in Aerospace Sciences*, pp. 19-46, 15 November 2015;

[11] L. Prandtl, "Applications of Modern Hydrodynamics," NACA, Göttingen University, 1923;

[12] H. Glauert, "Airplane propellers: division L," in *Aerodynamic theory IV,* Berlin, Springer, 1935, p. 169–360;

[13] R. Modarres, "Ideal Optimum Performance of Propellers, Lifting Rotors and Wind Turbines," in *All Thesis and Dissertations,* St. Louis, Washington University in St. Louis, 2013;

[14] Auld and Srinivas, "Blade Element Theory for Propellers," [Online]. Available: http://s6.aeromech.usyd.edu.au/aerodynamics/index.php/sample-page/propulsion/blade-element-propeller-theory/. [Accessed 25 May 2017].